A-1 Résolution d'équations du second degré

ÉNONCÉS DES EXERCICES

1-1. Résoudre les équations du second degré :

a) $x^2-5x+4=0$ **b)** $2x^2-4x+2=0$ **c)** $2x^2-x+4=0$

d) $3x^2-5x-8=0$ **e)** $5x^2-\frac{10}{7}x+\frac{5}{49}=0$ **f)** $3x^2+4x-4=0$

g) $5x^2-3x+15=0$ **h)** $2x^2-9x-5=0$ **i)** $x^2+8x-9=0$

j) $4x^2-12x+9=0$ **k)** $3x^2-10x+3=0$ **l)** $x^2-4x-21=0$

m) $x^2+2x+5=0$ **n)** $20x^2-9x+1=0$ **o)** $x^2+8x+12=0$

p) $-15x^2+11x-2=0$ **q)** $-x^2+4x-4=0$ **r)** $2x^2-7x+6=0$

1-2. Résoudre les équations du second degré :

a) $2x^2-10x+4=-2$ **b)** $2x^2-12x+16=-2$ **c)** $x^2+x=-1$

d) $(x+2)(x+3)=6$ **e)** $(x-15)(x+15)=400$ **f)** $x+\frac{1}{x-3}=5$

g) $\frac{9}{x}-\frac{x}{3}=2$ **h)** $\frac{6}{x}-\frac{6}{x+3}=\frac{1}{10}$ **i)** $\frac{x}{7}+\frac{21}{x+5}=\frac{47}{7}$

j) $\frac{750}{x}-\frac{720}{x-5}=-6$ **k)** $\frac{x}{x+4}+\frac{x}{x+1}-1=0$ **l)** $\frac{x}{x-1}-\frac{x+1}{x}=\frac{1}{2}$

m) $-\frac{1}{2}x^2+x=1$ **n)** $\frac{1}{\sqrt{3}}x^2-x=0$ **o)** $\frac{1}{4}x^2-\sqrt{2}x=2$

CORRIGÉS DES EXERCICES

1-1.

a) $\Delta = b^2 - 4ac = (-5)^2 - 4\times1\times4 = 25 - 16 = 9$. **Δ1**

$\Delta > 0$, il y a 2 solutions :

$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-(-5)+\sqrt{9}}{2\times1} = \frac{5+3}{2} = \frac{8}{2} = 4$ et **Δ2**

$x_2 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-(-5)-\sqrt{9}}{2\times1} = \frac{5-3}{2} = \frac{2}{2} = 1$

Donc $S = \{1;4\}$.

b) $\Delta = b^2 - 4ac = (-4)^2 - 4\times2\times2 = 16 - 16 = 0$. **Δ1**

$\Delta = 0$, il y a 1 solution double : $x_0 = \frac{-b}{2a} = \frac{-(-4)}{2\times2} = \frac{4}{4} = 1$. **Δ3**

Donc $S = \{1\}$.

c) $\Delta = b^2 - 4ac = (-1)^2 - 4\times2\times4 = 1 - 32 = -31$. **Δ1**

$\Delta < 0$, il n'y a pas de solution réelle. **Δ4**

Donc $S = \varnothing$.

d) $\Delta = b^2 - 4ac = (-5)^2 - 4\times3\times(-8) = 25 + 96 = 121$. **Δ1**

$\Delta > 0$, il y a 2 solutions :

$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-(-5)+\sqrt{121}}{2\times3} = \frac{5+11}{6} = \frac{16}{6} = \frac{8}{3}$ et **Δ2**

$x_2 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-(-5)-\sqrt{121}}{2\times3} = \frac{5-11}{6} = \frac{-6}{6} = -1$

Donc $S = \left\{-1;\frac{8}{3}\right\}$.

e) $\Delta = b^2 - 4ac = \left(-\frac{10}{7}\right)^2 - 4\times5\times\frac{5}{49} = \frac{100}{49} - \frac{100}{49} = 0$. **Δ1**

$\Delta = 0$, il y a 1 solution double : $x_0 = \frac{-b}{2a} = \frac{-\left(-\frac{10}{7}\right)}{2\times5} = \frac{\frac{10}{7}}{10} = \frac{1}{7}$. **Δ3**

Donc $S = \left\{\frac{1}{7}\right\}$.

f) $\Delta = b^2 - 4ac = 4^2 - 4\times3\times(-4) = 16 + 48 = 64$. **Δ1**

$\Delta > 0$, il y a 2 solutions :

$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-4+\sqrt{64}}{2\times3} = \frac{-4+8}{6} = \frac{4}{6} = \frac{2}{3}$ et **Δ2**

$x_2 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-4-\sqrt{64}}{2\times3} = \frac{-4-8}{6} = \frac{-12}{6} = -2$

Donc $S = \left\{-2; \frac{2}{3}\right\}$.

g) $\Delta = b^2 - 4ac = (-3)^2 - 4\times5\times15 = 9 - 300 = -291$. **Δ1**

$\Delta < 0$, il n'y a pas de solution réelle. **Δ4**

Donc $S = \varnothing$.

h) $\Delta = b^2 - 4ac = (-9)^2 - 4\times2\times(-5) = 81 + 40 = 121$. **Δ1**

$\Delta > 0$, il y a 2 solutions :

$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-(-9)+\sqrt{121}}{2\times2} = \frac{9+11}{4} = \frac{20}{4} = 5$ et **Δ2**

$x_2 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-(-9)-\sqrt{121}}{2\times2} = \frac{9-11}{4} = \frac{-2}{4} = -\frac{1}{2}$

Donc $S = \left\{-\frac{1}{2}; 5\right\}$.

i) $\Delta = b^2 - 4ac = 8^2 - 4\times1\times(-9) = 64 + 36 = 100$. **Δ1**

$\Delta > 0$, il y a 2 solutions :

$x_1 = \dfrac{-b+\sqrt{\Delta}}{2a} = \dfrac{-8+\sqrt{100}}{2\times1} = \dfrac{-8+10}{2} = \dfrac{2}{2} = 1$ et **Δ2**

$x_2 = \dfrac{-b-\sqrt{\Delta}}{2a} = \dfrac{-8-\sqrt{100}}{2\times1} = \dfrac{-8-10}{2} = \dfrac{-18}{2} = -9$

Donc $S = \{-9;1\}$.

j) $\Delta = b^2 - 4ac = (-12)^2 - 4\times4\times9 = 144 - 144 = 0$. **Δ1**

$\Delta = 0$, il y a 1 solution double : $x_0 = \dfrac{-b}{2a} = \dfrac{-(-12)}{2\times4} = \dfrac{12}{8} = \dfrac{3}{2}$. **Δ3**

Donc $S = \left\{\dfrac{3}{2}\right\}$.

k) $\Delta = b^2 - 4ac = (-10)^2 - 4\times3\times3 = 100 - 36 = 64$. **Δ1**

$\Delta > 0$, il y a 2 solutions :

$x_1 = \dfrac{-b+\sqrt{\Delta}}{2a} = \dfrac{-(-10)+\sqrt{64}}{2\times3} = \dfrac{10+8}{6} = \dfrac{18}{6} = 3$ et **Δ2**

$x_2 = \dfrac{-b-\sqrt{\Delta}}{2a} = \dfrac{-(-10)-\sqrt{64}}{2\times3} = \dfrac{10-8}{6} = \dfrac{2}{6} = \dfrac{1}{3}$

Donc $S = \left\{\dfrac{1}{3};3\right\}$.

l) $\Delta = b^2 - 4ac = (-4)^2 - 4\times1\times(-21) = 16 + 84 = 100$. **Δ1**

$\Delta > 0$, il y a 2 solutions :

$x_1 = \dfrac{-b+\sqrt{\Delta}}{2a} = \dfrac{-(-4)+\sqrt{100}}{2\times1} = \dfrac{4+10}{2} = \dfrac{14}{2} = 7$ et **Δ2**

$x_2 = \dfrac{-b-\sqrt{\Delta}}{2a} = \dfrac{-(-4)-\sqrt{100}}{2\times1} = \dfrac{4-10}{2} = \dfrac{-6}{2} = -3$

Donc $S = \{-3;7\}$.

m) $\Delta = b^2 - 4ac = 2^2 - 4\times1\times5 = 4-20 = -16$. **Δ1**

$\Delta < 0$, il n'y a pas de solution réelle. **Δ4**

Donc $S = \varnothing$.

n) $\Delta = b^2 - 4ac = (-9)^2 - 4\times20\times1 = 81-80 = 1$. **Δ1**

$\Delta > 0$, il y a 2 solutions :

$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-(-9)+\sqrt{1}}{2\times20} = \frac{9+1}{40} = \frac{10}{40} = \frac{1}{4}$ et **Δ2**

$x_2 = \frac{-b-\sqrt{\Delta}}{2a} == \frac{-(-9)-\sqrt{1}}{2\times20} = \frac{9-1}{40} = \frac{8}{40} = \frac{1}{5}$

Donc $S = \left\{\frac{1}{4};\frac{1}{5}\right\}$.

o) $\Delta = b^2 - 4ac = 8^2 - 4\times1\times12 = 64-48 = 16$. **Δ1**

$\Delta > 0$, il y a 2 solutions :

$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-8+\sqrt{16}}{2\times1} = \frac{-8+4}{2} = \frac{-4}{2} = -2$ et **Δ2**

$x_2 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-8-\sqrt{16}}{2\times1} = \frac{-8-4}{2} = \frac{-12}{2} = -6$

Donc $S = \{-6;-2\}$.

p) $\Delta = b^2 - 4ac = 11^2 - 4\times(-15)\times(-2) = 121-120 = 1$. **Δ1**

$\Delta > 0$, il y a 2 solutions :

$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-11+\sqrt{1}}{2\times(-15)} = \frac{-11+1}{-30} = \frac{-10}{-30} = \frac{1}{3}$ et **Δ2**

$x_2 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-11-\sqrt{1}}{2\times(-15)} = \frac{-11-1}{-30} = \frac{-12}{-30} = \frac{12}{30} = \frac{2}{5}$

Donc $S = \left\{\frac{1}{3};\frac{2}{5}\right\}$.

q) $\Delta = b^2 - 4ac = 4^2 - 4\times(-1)\times(-4) = 16 - 16 = 0$. **Δ1**

$\Delta = 0$, il y a 1 solution double : $x_0 = \frac{-b}{2a} = \frac{-4}{2\times(-1)} = \frac{-4}{-2} = 2$. **Δ3**

Donc $S = \{2\}$.

r) $\Delta = b^2 - 4ac = (-7)^2 - 4\times2\times6 = 49 - 48 = 1$. **Δ1**

$\Delta > 0$, il y a 2 solutions :

$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-(-7)+\sqrt{1}}{2\times2} = \frac{7+1}{4} = \frac{8}{4} = 2$ et **Δ2**

$x_2 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-(-7)-\sqrt{1}}{2\times2} = \frac{7-1}{4} = \frac{6}{4} = \frac{3}{2}$

Donc $S = \left\{\frac{3}{2};2\right\}$.

1-2.

a) $2x^2 - 10x + 4 = -2 \Leftrightarrow 2x^2 - 10x + 6 = 0$.

$\Delta = b^2 - 4ac = (-10)^2 - 4\times2\times6 = 100 - 48 = 52$. **Δ1**

$\Delta > 0$, il y a 2 solutions :

$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-(-10)+\sqrt{52}}{2\times2} = \frac{10+2\sqrt{13}}{4} = \frac{5+\sqrt{13}}{2}$ et **Δ2**

$x_2 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-(-10)-\sqrt{52}}{2\times2} = \frac{10-2\sqrt{13}}{4} = \frac{5-\sqrt{13}}{2}$

Donc $S = \left\{\frac{5-\sqrt{13}}{2};\frac{5+\sqrt{13}}{2}\right\}$.

b) $2x^2 - 12x + 16 = -2 \Leftrightarrow 2x^2 - 12x + 18 = 0$.

$\Delta = b^2 - 4ac = 12^2 - 4\times2\times18 = 144 - 144 = 0$. **Δ1**

$\Delta = 0$, il y a 1 solution double : $x_0 = \frac{-b}{2a} = \frac{-(-12)}{2\times2} = \frac{12}{4} = 3$. **Δ3**

Donc $S=\{3\}$.

c) $x^2+x=-1 \Leftrightarrow x^2+x+1=0$

$\Delta=b^2-4ac=1^2-4\times1\times1=1-4=-3$. **Δ1**

$\Delta<0$, il n'y a pas de solution réelle. **Δ4**

Donc $S=\varnothing$.

d) $(x+2)(x+3)=6 \Leftrightarrow x^2+5x+6=6 \Leftrightarrow x^2+5x=0 \Leftrightarrow x(x+5)=0$

$\begin{cases} x=0 \\ \text{ou} \\ x+5=0 \end{cases} \Leftrightarrow \begin{cases} x=0 \\ \text{ou} \\ x=-5 \end{cases}$. Donc $S=\{-5;0\}$. **Δ4**

e) $(x-15)(x+15)=400 \Leftrightarrow x^2-15^2=400 \Leftrightarrow x^2-225=400 \Leftrightarrow x^2=625$

$\Leftrightarrow \begin{cases} x=\sqrt{625} \\ \text{ou} \\ x=-\sqrt{625} \end{cases} \Leftrightarrow \begin{cases} x=25 \\ \text{ou} \\ x=-25 \end{cases}$. Donc $S=\{-25;25\}$.

f) $x+\dfrac{1}{x-3}=5 \Leftrightarrow \dfrac{x(x-3)}{x-3}+\dfrac{1}{x-3}=\dfrac{5(x-3)}{x-3} \Leftrightarrow x(x-3)+1=5(x-3)$

$\Leftrightarrow x^2-3x+1=5x-15 \Leftrightarrow x^2-8x+16=0$

$\Delta=b^2-4ac=(-8)^2-4\times1\times16=64-64=0$. **Δ1**

$\Delta=0$, il y a 1 solution double : $x_0=\dfrac{-b}{2a}=\dfrac{-(-8)}{2\times1}=\dfrac{8}{2}=4$. **Δ3**

Donc $S=\{4\}$.

g) $\dfrac{9}{x}-\dfrac{x}{3}=2 \Leftrightarrow x\times\left(\dfrac{9}{x}-\dfrac{x}{3}\right)=x\times2 \Leftrightarrow x\times\dfrac{9}{x}-x\times\dfrac{x}{3}=2x$

$\Leftrightarrow 9-\dfrac{x^2}{3}-2x=0 \Leftrightarrow -\dfrac{x^2}{3}-2x+9=0$

$\Delta=b^2-4ac=(-2)^2-4\times\left(-\dfrac{1}{3}\right)\times9=4+12=16$. **Δ1**

$\Delta>0$, il y a 2 solutions :

CORRIGÉS DES EXERCICES

$x_1=\dfrac{-b+\sqrt{\Delta}}{2a}=\dfrac{-(-2)+\sqrt{16}}{2\times\left(-\dfrac{1}{3}\right)}=\dfrac{2+4}{-\dfrac{2}{3}}=\dfrac{6}{-\dfrac{2}{3}}=6\times\left(-\dfrac{3}{2}\right)=-9$ et

$x_2=\dfrac{-b-\sqrt{\Delta}}{2a}=\dfrac{-(-2)-\sqrt{16}}{2\times\left(-\dfrac{1}{3}\right)}=\dfrac{2-4}{-\dfrac{2}{3}}=\dfrac{-2}{-\dfrac{2}{3}}=-2\times\left(-\dfrac{3}{2}\right)=3$

Donc $S=\{-9;3\}$.

h) $\dfrac{6}{x}-\dfrac{6}{x+3}=\dfrac{1}{10}\Leftrightarrow\dfrac{6(x+3)}{x(x+3)}-\dfrac{6x}{x(x+3)}=\dfrac{1}{10}\Leftrightarrow\dfrac{6(x+3)-6x}{x(x+3)}=\dfrac{1}{10}$

$\Leftrightarrow\dfrac{18}{x(x+3)}=\dfrac{1}{10}\Leftrightarrow 18\times10=x(x+3)\Leftrightarrow 180=x^2+3x\Leftrightarrow x^2+3x-180=0$

$\Delta=b^2-4ac=3^2-4\times1\times(-180)=9+720=729$. **Δ1**

$\Delta>0$, il y a 2 solutions :

$x_1=\dfrac{-b+\sqrt{\Delta}}{2a}=\dfrac{-3+\sqrt{729}}{2\times1}=\dfrac{-3+27}{2}=\dfrac{24}{2}=12$ et

$x_2=\dfrac{-b-\sqrt{\Delta}}{2a}=\dfrac{-3-\sqrt{729}}{2\times1}=\dfrac{-3-27}{2}=\dfrac{-30}{2}=-15$

Donc $S=\{-15;12\}$.

i) $\dfrac{x}{7}+\dfrac{21}{x+5}=\dfrac{47}{7}\Leftrightarrow\dfrac{x(x+5)}{7(x+5)}+\dfrac{21\times7}{7(x+5)}=\dfrac{47(x+5)}{7(x+5)}$

$\Leftrightarrow x(x+5)+147=47(x+5)\Leftrightarrow x^2+5x+147=47x+235$

$\Leftrightarrow x^2-42x-88=0$

$\Delta=b^2-4ac=(-42)^2-4\times1\times(-88)=2116$. **Δ1**

$\Delta>0$, il y a 2 solutions :

$x_1=\dfrac{-b+\sqrt{\Delta}}{2a}=\dfrac{-(-42)+\sqrt{2116}}{2\times1}=\dfrac{42+46}{2}=\dfrac{88}{2}=44$ et

$x_2=\dfrac{-b-\sqrt{\Delta}}{2a}=\dfrac{-(-42)-\sqrt{2116}}{2\times1}=\dfrac{42-46}{2}=\dfrac{-4}{2}=-2$

Donc $S=\{-2;44\}$.